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Improvement of Quantum Circuits Using H-U-H Sandwich Technique with Diagonal Matrix Implementation

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ABSTRACT. Quantum circuits are ideal for most of the modern world problems. They are more efficient and reliable for many computations that are a challenge for classical computers. Beside this they are themselves more complex to deal with and too much costly to build. Cost is the most crucial factor for designing or implementation of any circuit but when it comes to quantum circuits it becomes the most unavoidable part of design technique. The hadamard-unitary sandwich technique has an amazing cost reduction potential in quantum circuits. In this paper we have discussed about how H-U-H sandwich technique came up with a break through minimizing the circuit complexity and played leading role in cost reduction of quantum circuits by minimizing number of gates used. This method also helps in achieving high computation power and efficiency with more feasibility.

Keywords: Quantum circuit; H-U-H sandwich technique; Diagonal matrix; Quantum computing.

INTRODUCTION

Quantum computing is auspicious strategy to simulate such systems that cannot be done by classical computers.[1, 2] Quantum circuits are much powerful and can solve some certain problems with great ease in very less time such as factoring integers[3] whereas super polynomial classical algorithm solved by using quantum Shor's algorithm[4] can attack on RSA cryptosystem.[5] Quantum circuit can be defined as a set of quantum gates performing computations by taking inputs in qubits and generating outputs. These quantum gates are used to perform different type of computations and are interconvertible and replaced with some another combination to perform same functions. If any Unitary Transformation quantum data can be efficiently approximated arbitrarily well as a sequence of gates in the set then we say the set of quantum gates to be universal.[6] These gates can easily be transpiled by using C-NOT (Cx) gates and hadamard gates with single qubit rotations. These forms universal gate set as well as any unitary can be implemented using them. The term unitary used here is basically a complex analog of real orthogonal matrix. It can be defined as a matrix whose inverse equals to its conjugate transpose. Basically it is necessary to form an efficient unitary matrix that can describe quantum algorithms and can be decomposed into known quantum gates.[7, 8]

Any quantum circuit (any at all!) can be represented as a single unitary transformation. This decomposition of unitary matrix into quantum gates serves as an optimization problem. In quantum circuits mostly the gates are replaceable with one another. When the matrices of two contagious gates are commute we can easily replace gates with another one. Mostly we use it in the optimization of reversible gates. The two contagious gates are replaceable either they are not applied on same qubit or they are T, S, S†, T†. C-Not gates do not support optimization directly and mapping of these gates come up with an added cost in the circuits.[9, 10] In quantum circuit's transformation, mostly we have to face two major resistances: complex and huge search spaces that are itself an open challenge as well as in circuit simulation high cost becomes unavoidable too.[7] This complexity and high cost is mostly due to excessive number of quantum gates in the circuit which can be formed by lesser number of gates

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just by getting the best choice of gates and preferably the favorable sequencing of these gates. The above discussed favorable sequencing can also be attained by using hadamard gates. It also helps to reduce T-depths as well as T-counts in quantum circuits.[9] We used hadamard gate as it is its own inverse, so applying H-H anywhere in a circuit won't change the circuit. The whole reason to do unitary-hadamard multiplication is that the transpiler is much better at optimizing the transformed circuit than the original ones. Circuit decomposition and transformation in this manner leads to fast compilation and rapid data travelling. So the decomposed smaller circuits run more efficiently and rapidly than the larger ones with the same output.

Diagonal alignment of circuit matrices before decomposition will be an addition to the computational power to the circuits. One of the objectives of these matrices includes generation of random states which can be utilized in long range of quantum applications. These includes efficient measurements of qubits, gates fidelities estimation etc. Just because these random states uses exponential resources therefore the possibilities also increases exponentially as well as it increases no. of qubits. Quantum gates set have their own matrices that are somehow interchangeable. So in this paper we will discuss the matrices of quantum circuits and will perform operations on them to form a better form of diagonal matrix so that we can achieve their random states and can minimize the original circuit to very lesser number of possible quantum gates. This approach will lead to higher efficiency and huge decrement in circuit costs.

Our study will be proceeding with getting a unitary matrix and transforming it by using HUH sandwich technique. We will form a diagonal matrix from the HUH sandwich matrix to increase computational power. Later on the matrix will be ready to decompose so that it will be able to implement by using lesser number of quantum gates. This method will be less complex and more cost effective as well as having more computation power and efficiency too in comparison with other methods.

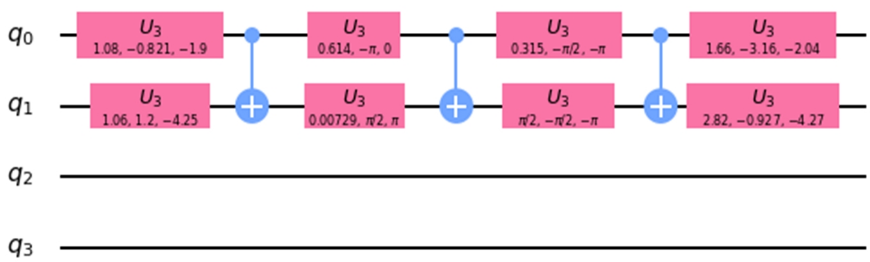

Fig. 1: Decomposed unitary(4) circuit.

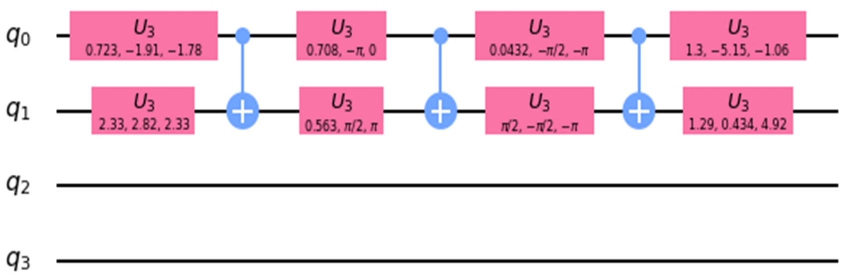

Fig. 2: H.U decomposed circuit.

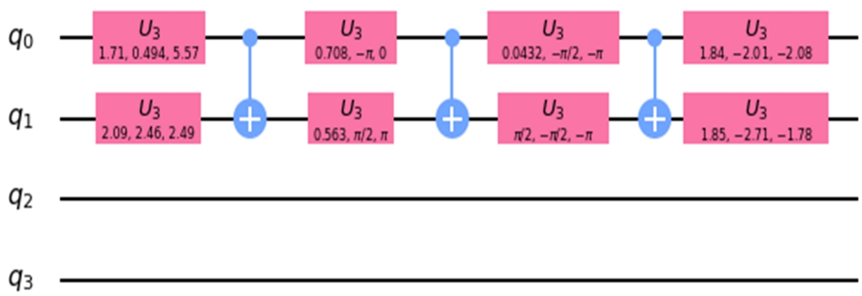

Fig. 3: H.U.H decomposed circuit.

MATERIALS AND METHODS

H-U-H Sandwich Technique

Hadamard gate is known for the superposition of qubits which leads to the more computation power which is major factor of quantum circuits. We look for dot product of hadamard matrix and unitary, we come up with unitary qubit rotations. We can observe the circuits in Figs. 1-3. Fig. 1 shows the original circuit of 4X4 unitary matrix. Where, U3 is representing hadamard gate while other one is representation of C-Not gate. Fig. 2 is a transformed unitary matrix shown in Fig. 1 formed by dot product of hadamard matrix with unitary matrix. Fig. 3 is a transformed unitary matrix formed by the dot product of hadamard-Unitary matrix shown in Fig. 2 with hadamard matrix, thus forming H-U-H sandwich. These three circuits shows the qubit rotations due to this sandwich technique adding more computation power to the circuit and due to this the circuit becomes more efficient and fast as well.

Through this approach we managed to minimize the circuit more than 60% of the original circuit. The main logic behind this approach is to sandwich our unitary between two hadamard matrices. Then we managed to find a diagonal matrix out of this sandwich. This diagonal matric on decomposition forms a quantum circuit having more than 60% lesser number of quantum gates than in the original unitary circuit. Here we have used qiskit by IBM for quantum circuit simulations.[9] Methodology proceeds in the steps described as below.

Forming Unitary Matrix

First, we will form a unitary matrix of 4X4 by the scipy function. It will generate a random unitary matrix. Fig. 4 shows the random 4X4 matrix formed by importing unitary_group from stats of Python library scipy.

Transformation of Unitary by H-U-H

The above formed unitary matrix transformed into H-U-H sandwich by taking dot products of hadamard with unitary and with hadamard. Fig. 5 shows the transformed H-U-H matrix by having dot product of Hadamard matrix with unitary matrix and again with Hadamard matrix.

Conversion of H-U-H into its Diagonal Matrix

Diagonal matrix has great importance in mathematical calculations. Here we converted the efficient H-U-H matrix into its diagonal matrix to minimize the quantum gates required in original circuit, after diagonalization we got the diagonalized matrix shown in Fig. 6.

Decomposition and Transpilation of Diagonal Matrix

Above discussed diagonal matrix is then decomposed to form quantum circuits. After transpilation (source-to-source compilation) of decomposed matrix we got the minimized circuit of 3 hadamard and 2 C-Not gates as represented in Fig. 7 below, which are much lesser than the original unitary matrix circuit in Fig. 1 having 6 hadamard gates along with 3 C-Not gates.

```
In [108]:
from scipy.stats import unitary_group
u = unitary_group.rvs(4)
print(u)

[[-0.03365911-0.58466424j -0.02611549+0.31396476j -0.57410203+0.40385396j
   0.08781724+0.23953263j]
 [-0.10251499-0.41854486j -0.10601577-0.83460001j  0.10598038+0.02871662j
  -0.10277852+0.28964539j]
 [-0.14852849+0.23516372j -0.06549475-0.38091685j -0.57042652+0.1885509j
   0.00622702-0.64208526j]
 [ 0.26719672+0.5679691j   0.05966817-0.20000024j -0.21575203+0.29625254j
   0.36803237+0.54101229j]]
```

Fig. 4: Transformed H.U.H matrix.

```
In [135]:
import numpy as np
u1 = np.dot(h, u)
u2 = np.dot(u1, h)
print(u2)

[[-0.26261645+0.01096262j -0.37328658+0.34768626j  0.18488459-0.66177692j
   0.43351257+0.10305177j]
 [-0.39952462-0.12426281j -0.2912885 +0.00851827j  0.20345863+0.3586245j
   0.14048515-0.74180473j]
 [-0.1130777 -0.29201049j  0.14468498-0.6372769j  -0.07749581-0.58101955j
  -0.20895381-0.29603498j]
 [ 0.22915937+0.77799778j -0.14957279-0.4532353j   0.11566277-0.02991409j
   0.28933173-0.12816239j]]
```

Fig. 5: H.U.H diagonal matrix.

```
In [137]:
d = []
for i in range (4):
    d.append(u2[i,i])
print(d)

[(-0.26261644754264224+0.010962615875947546j), (-0.2912885001998185+0.008518272660881454j), (-0.07749580668
789649-0.5810195496847139j), (0.28933172799730006-0.12816239290151893j)]
```

Fig. 6: H.U.H diagonal matrix.

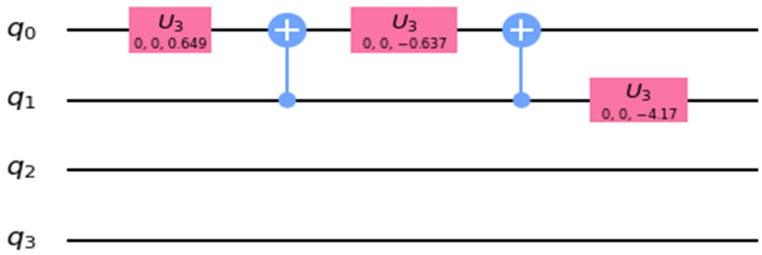

Fig. 7: Decomposed H.U.H diagonal matrix circuit.

RESULTS AND DISCUSSION

Study Analytics

From the discussed case study, we can analyze that, in our case the major problem was the huge complex circuit with multiple number of gates. It can be much more critical when it comes to practical implementation of the circuit. To solve this we formed a new reduced form of the previous circuit with much lesser number of gates. In our case we formed 4X4 unitary matrix through 8 hadamard gates accompanied with 3 C-Not gates. After implementation of discussed method we successfully reduced the same circuit and got it with 3 hadamard gates along with 2 C-Not gates. It will be more and more beneficial for the circuits with larger number of gates. We used this method to analyze bigger circuits e.g. for 8X8, 16X16, 32X32, and so on. The analytics for larger circuits were more amazing then we expected because shows drastically exponential reduction in this case.

Graphs of Figs. 8-10 are schematically showing the comparison of no. of gates used in the circuit for the formation of unitary matrix before and after implementation of H-U-H sandwich technique. Fig. 8 is showing exponential to linear reduction of C-Not gates used in original circuit as compared to transformed unitary circuit. Fig. 9 is showing all about hadamard gates used in the circuit. In contrast, Fig. 10 is describing the collectively usage of no. of gates for the implementation of unitary matrix to quantum circuits in original to transformed circuit.

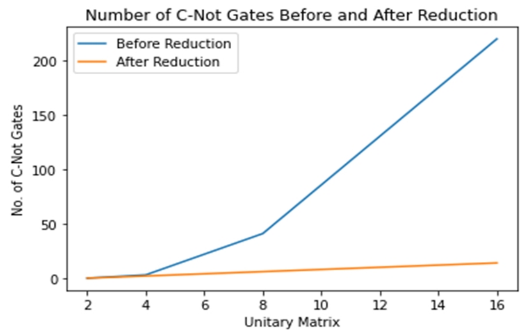

Fig. 8: Comparison of no. of C-Not gates before and after reduction.

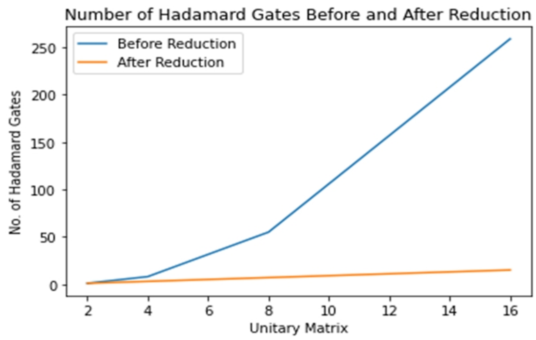

Fig. 9: Comparison of no. of hadamard gates before and after reduction.

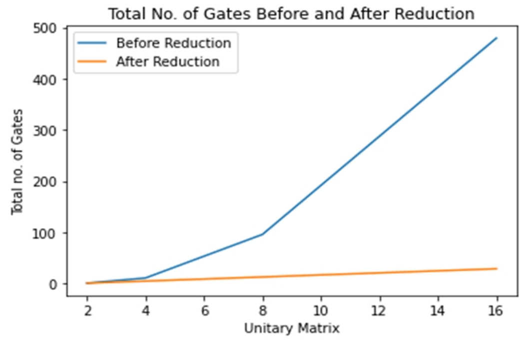

Fig. 10: Comparison of total no. of gates used in the circuit before and after reduction.

Study Findings

During this study we applied this technique to number of square unitary matrices and came up with the result that it can be applied to 2^n unitary matrices. Where, n is number of qubits ranging from 1 to infinity. But for circuit simulation we could only simulate the circuit up to 2^6 unitary matrices. The reason behind this constraint was the limitation of number of pixels that it must be less than 2^16 in each direction but our case the image size were too much larger than the limit.

For big picture some snaps of the different unitary circuits before and after optimizations are provided

a) 2X2 unitary matrix: Figs. 11 and 12 show the decomposed 2X2 unitary matrix in comparison with transformed diagonal matrix.

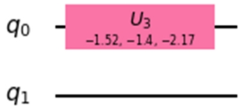

Fig. 11: Decomposed unitary(2) matrix.

Fig. 12: Decomposed H.U.H diagonal 2X2 matrix circuit.

b) 8X8 unitary matrix: Fig. 13 and 14 explain the observed drastic change as the circuit becomes larger with multiple no. of gates. These results are also applicable to 16X16, 32X32, and 64X64 matrices and so on. Amazingly, efficiency and reduction ratio increase drastically on higher dimensions.

Fig. 13: Decomposed unitary(8) matrix.

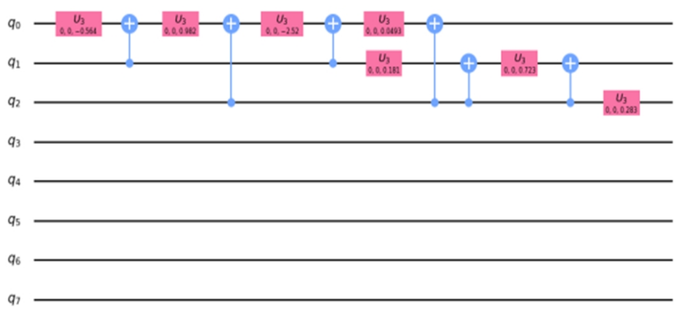

Fig. 14: Decomposed H.U.H diagonal 8X8 matrix circuit.

So we come up with the result that H-U-H sandwich technique made the circuit efficient with more computation power. While, diagonalization reduced the number of quantum gates. H-U-H sandwich method works like a magic for the most complex circuits with hundreds or thousands of quantum gates in it. The decrement in number of gates in the optimized circuits, compared to the original unitary circuits increase exponentially in the larger circuits with greater number of quantum gates. Due to great decrement of quantum gates in the circuit, circuit cost is reduced to great extent and this reduction will also reduce circuit complexity and will add to efficiency too. This method will specifically work on orthonormal matrices having isometry. So during this optimization we must have to take care about the orthonormality of matrix.

CONCLUSION

Quantum circuits are highly efficient to compute the tasks which take exponentially much more time and power to perform in classical computers. But the fact is quantum circuits itself are more complex and come up with huge cost. That is one of the major problems so that quantum circuits are not more practically in use. Reason behind our study was to reduce this high complexity by minimizing the required number of gates used in the formation of a specific circuit. To do this we applied hadamard gate matrix and some mathematical calculations. Finally, we concluded that our H-U-H sandwich technique along diagonal matrix formation could result in high computation power by achieving random qubit states, less complexity with higher efficiency of circuits. Along with all these minimizing the number of gates in circuit itself reduced the per quantum gate costs involved in circuits as an advantage regarding the economic situation.

REFERENCES

1. Burns J, Sung Y, Wight C. Decomposing the hamiltonian of quantum circuits using machine learning. Phys. Caps. Proj. 2019; 80.
2. Lenstra AK, Lenstra HW, Manasse MS, Pollard JM. The number field sieve. Proc. 22nd Ann. ACM Symp. Theor. Comput. 1990;14-16.
3. Shor PW. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. SIAM Rev. 1999;41:303-332.
4. Nam Y, Ross NJ, Su Y, Childs AM, Maslov D. Automated optimization of large quantum circuits with continuous parameters. Quant. Inf. 2018;4:1-2.
5. Asfaw A, Bello L, Ben-Haim Y, Bravyi S, Capelluto L, Vazquez AC, Ceroni J, Gambetta J, Garion S, Gil L, Gonzalez SD. Learn quantum computation. Qiskit Text. 2019.
6. Daskin A, Kais S. Decomposition of unitary matrices for finding quantum circuits: Application to molecular hamiltonians. J. Chem. Phys. 2011;134:144112.
7. Nielsen MA, Chuang IL, Chuang IL. Quantum computation and quantum information. Cambridge University Press; 2000.
8. Dueck GW, Pathak A, Rahman MM, Shukla A, Banerjee A. Optimization of circuits for IBM's five-qubit quantum computers. IEEE 21st Euromic. Conf. Dig. Sys. Des. 2018;680-684.
9. Maslov D, Dueck GW, Miller DM. Toffoli network synthesis with templates. IEEE Transact. Comput. Aid. Des. Integrat. Circ. Sys. 2005;24:807-817.
10. Abdessaied N, Soeken M, Drechsler R. Quantum circuit optimization by hadamard gate reduction. Int. Conf. Rev. Comput. 2014;149-162.

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