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Suppressing Chaotic Oscillations of a Spherical Cavitation Bubble by Slave-Master Feedback

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ABSTRACT. Dynamics driven a single bubble is known to be a complex phenomenon indicative of a highly active nonlinear as well as chaotic behavior. Based on theoretical aspects, so much information are available in this case. Within this current research work, a method based on Slave-Master Feedback (SMF) to suppress chaotic oscillations was introduced. In the Slave-Master Feedback control process, the spherical cavitation bubble as the slave system is coupled with a dynamical system as the master, so its implementation becomes quite simple and similar statements can be made for the high dimensional cases. In order, we perturbed the fundamental acoustic energy by applying the proposed technique. A great virtue of this method is its flexibility. Also, unlike other chaos control techniques, there is no need to know more than one variable. The problem of the transition to chaos in deterministic systems has been the subject of much interest, and, for low dimensional dynamics, it was found that the transition most often occurs via a small number of often observed routes. The relation between this method and frequency ultrasonic irradiation is correlated to prove its applicability in applications involving cavitation phenomena. The results indicated its strong impact on reducing the chaotic oscillations to regular ones. Due to the importance of topic in various aspects, investigation of the efficacy of the slave-master feedback control method in a system of interacting bubbles could be one of the subjects for future studies.

Keywords: Chaotic; Oscillation; Bubble.

INTRODUCTION

Mathematically, all nonlinear dynamical systems with more than two degrees of freedom can display chaos and, therefore, become unpredictable over longer time scales. The problem of the transition to chaos in deterministic systems has been the subject of much interest, and, for low dimensional dynamics, it has been found that this transition most often occurs via a small number of often observed routes (e.g., period doubling and intermittency). A gas bubble driven in motion by ultrasound is an example of a system with highly nonlinear properties in which deterministic chaos manifests itself. This phenomenon occurs when a high amplitude, high frequency sound is applied to the liquid.[1-3] The emergence of cavitation bubble structures is a common feature of ultrasound applications in liquids. The study of their behavior has recently been the center of attention. It is believed that this phenomenon exhibits highly complex and chaotic dynamics both experimentally[4-6] and numerically.[7-9] One of the key factors in cavitation bubble dynamics is bubble-bubble interaction. bubble-bubble and bubble-fluid interaction play an important role in a number of natural phenomena, such as sound propagation in the ocean, the exchange of gases and heat between the oceans and the atmosphere, and explosive volcanic eruptions. After the generation of the cavitation bubble in the liquid they begin their nonlinear oscillations.

In recent years, the modern methods of nonlinear dynamical systems analysis have led to a substantial improvement in understanding of the nonlinear behavior of bubbles and clusters of bubbles.[10-12] The destructive nature of cavitation bubble is widely reported in literature, such as hydrodynamic cavitation, shock wave lithotripsy, sonofusion, in material science,

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sonoluminescence and sonochemistry.[6, 13-20] Macdonald et al.[21] analyzed the bifurcation structure of isolated and interacting encapsulated microbubbles as used in medical diagnostic imaging. So, it is important to establish methods to study the bubble radial stability in different conditions. In the above mentioned applications, an optimum employment of bubbles entails that chaotic oscillations be reduced, because when the bubble motion gets chaotic, its behavior becomes unpredictable and very hard to deal with. Therefore, reducing chaotic oscillations could be the first step in controlling the bubble dynamics by providing more accurate predictions.

The main argument of this paper is the stability analysis of a spherical cavitation bubble and the selection of most suitable control parameter for establishing an adequate control strategy based on the slave-master feedback.[22] In applications involving cavitation, the parameters like the viscosity, surface tension or the diameter of the bubble are determined by the media and type of the application. Consequently, the only remaining parameter that is to perturb for stabilizing the motion is the forcing term. Hence, parameter such as pressure is used as control parameter to obtain different dynamical regimes through plotting the Lyapunov exponent spectra, bifurcation diagrams. After selection the frequency as suitable control parameter of the system, this parameter as a discrete time dynamical system is coupled with the bubble.

MATERIALS AND METHODS

The Bubble Model

The bubble model used for the numerical simulation was derived,[7] which is a modified model of Keller-Miksis equation[23] formulated by Prosperetti[24] and is given by eq. (1):

$$(1-\frac{\dot{R}}{c})R\ddot{R}+\frac{3}{2}\dot{R}^2(1-\frac{\dot{R}}{3c})=(\frac{P}{\rho})+\frac{1}{\rho c}\frac{d}{dt}(RP)=(1+\frac{\dot{R}}{c})\frac{P}{\rho}+\frac{R}{\rho c}\frac{dP}{dt} \quad (1)$$

with

$$P(R,\dot{R},t)=(P_{stat}-P_{\nu}+\frac{2\sigma}{R_0})(\frac{R_0}{R})^{3k}-\frac{2\sigma}{R}-4\mu\frac{\dot{R}}{R}-P_{stat}+P_{\nu}-P_a\sin(2\pi f t)$$

In this equation R is the bubble radius, R_0 is the initial radius, $\dot{R}$ is the bubble velocity, $\ddot{R}$ is the bubble wall acceleration, f is the frequency of the driving sound field, P_a is the amplitude of the driving pressure, P_{stat} is the static ambient pressure, P_v is the vapor pressure, σ is the surface tension, ρ is the liquid density, μ is the viscosity, c is the sound velocity, and k is the polytropic exponent of the gas in the bubble. The model was solved for bubble using the values of the physical constants shown in Table. 1. Between control parameters Pa and R0 are the most important ones and the correspondent values are stated.

Stability Analysis

The presented model has the capability to be used for determining the behavior of a bubble. However the effects of parameters mentioned above are studied for a bubble in a wide range of parameter domain. Through analyzing the results more comprehensive knowledge would be available about the complex and nonlinear dynamics of bubble. For stability analysis, it is convenient to transform the second-order differential eq. (1) into an autonomous system of first-order differential eq. (2) of the following form:[21, 25]

$$\dot{B}(x,y,\theta)=\begin{cases}\dot{x}=y\\ \dot{y}=[-\frac{y^2}{2}(3-\frac{y}{c})+(1+(1-3k)\frac{y}{c})(\frac{P_{stat}-P_{\nu}}{\rho}+\frac{2\sigma}{\rho x_0})(\frac{x_0}{x})^{3k}-\frac{2\sigma}{\rho R}-4\mu\frac{y}{\rho x}\\ \quad -(1-\frac{y}{c})(\frac{P_{stat}-P_{\nu}+P_a\sin(2\pi\theta)}{\rho})-x\frac{2\pi f P_s}{\rho c}\cos(2\pi\theta)][(1-\frac{y}{c})x+\frac{4\mu}{\rho c}]^{-1}\\ \dot{\theta}=f\end{cases} \quad (2)$$

or equivalently:

$$\frac{dV}{dt}=F(V,\alpha) \quad (3)$$

where θ is the cyclic variable, $V(x, y, \theta)$ an autonomous vector field and $\alpha(R_0, P_a, P_{stat}, P_v, v, \mu, \rho, c, k)$ is an element of the parameter space. This system generates a flow $\Phi=\{\Phi^T\}$ on the phase space $M=R^2*S$ and there exist a global map:

$$P:\sum_c \longrightarrow \sum_c$$
$$V_P(x,y,\theta)\longrightarrow P(V_P)=\{\Phi^T\}|_{\sum_c(x,y,\theta_0)}$$

with $T=\frac{1}{\nu}$, θ_0 is a constant determining the Poincare cross-section and (x, y) the coordinates of the attractors in the Poincare cross-section $\sum_c$, which is defined by:

$$\sum_c=\{(x,y,\theta)\in R^2*S^1|\theta=\theta_0\}$$

The choice of Poincare section is arbitrary; the only necessary condition is that the trajectory should cross the section once every acoustic cycle. For driven oscillators like the bubble model, a natural way to define $\sum$ is to cut the torus like state space M transversally to the cyclic θ direction at a fixed value θ_0 of θ.[7] In this paper the stability of a single cavitation bubble is studied versus the driving pressure amplitude.

Slave-Master Feedback Control

The questions of chaos control are actively discussed in scientific and technical fields.[26-28] It is appropriate to decompose the applied works on chaos into scientific and technical (engineering) applications. Most recently, the following methods of control have been proven to be successful in the experimental control of chaos:

- Determination of the stable and unstable directions in the Poincare section.
- Self-controlling feedback procedure.
- Introduction of small modulation of a control parameter.
- Knowledge of a prescribed goal dynamics.

The first two methods are usually called feedback methods, while the third and last are called non-feedback methods.[26, 29] The third method, small modulation, theoretically has focused on the suppression of chaos in the dynamics of different models. Although control of chaos by small modulations has not been proved in general,[30] this method includes chaotic behavior generated by error signals due to the difference between the output signal and its value at an earlier time.

The present analysis consider the parameter *f* (frequency) to be variable in the time such that thoroughly be change by another chaotic map. By considering a spherical cavitation bubble and the control system as a two dimensional dynamical system, the simple model for controlling the stability of a spherical cavitation bubble is introduced as follows:

$$\dot{B}(x,y,\theta,f)=\begin{cases}\dot{B}(x,y,\theta)=\begin{cases}\dot{x}=y\\ \dot{y}=[-\frac{y^2}{2}(3-\frac{y}{c})+(1+(1-3k)\frac{y}{c})(\frac{P_{stat}-P_\nu}{\rho}+\frac{2\sigma}{\rho x_0})(\frac{x_0}{x})^{3k}\\ \quad -\frac{2\sigma}{\rho R}-4\mu\frac{y}{\rho x}\ -(1-\frac{y}{c})(\frac{P_{stat}-P_\nu+P_a\sin(2\pi\theta)}{\rho})\\ \quad -x\frac{2\pi f P_s}{\rho c}\cos(2\pi\theta)][(1-\frac{y}{c})x+\frac{4\mu}{\rho c}]^{-1}\end{cases}\\ \dot{f}=\frac{4f-(1-f)^2}{\xi_t(1-f)^2}\end{cases} \tag{4}$$

Where ξ_t is an arbitrary number between (0, 1). The concept of feedback control has been re-defined by taking into account the control parameter as a variable in the time that is changeable by another chaotic map, for which a new and effective control scheme has been presented. As this method is independent of geometrical considerations, so that, it can easily be applied in high dimensional dynamical systems. So, it makes it possible for us to study the interaction of the bubbles by a controlled technique in which we can realize the bubble cluster better. We describe a method for dynamical control of chaos in a spherical cavitation bubble. The chaos control problem has been studied without requiring any knowledge about the state of system.

Analysis Tool

There are several mathematical tools available for quantifying bubble stability ranging, the reasons to use maximum Lyapunov exponents and bifurcation structure in the absence of direct mathematical methods are:

- The maximum Lyapunov exponents, approximated computationally for a wide range of injection values, indicates clearly the chaotic behavior of bubble interaction dynamics.
- The computationally based bifurcation analysis shows that the bubble interaction dynamics transits among different regions such as fixed point, chaotic attractors and intermittent behavior.

Lyapunov Exponent Spectrum

Lyapunov exponents and entropy measures, can be considered as “dynamic” measures of attractors complexity and are called “time average”.[31] The Lyapunov exponent λ is useful for distinguishing various orbits. The Lyapunov exponents quantify sensitivity of the system to initial conditions and give a measure of predictability. The Lyapunov exponents are a measure of the rate at which the trajectories separate one from another. A negative exponent implies that the orbits approach to a common fixed point. A zero exponent means that the orbits maintain their relative positions; they are on a stable attractor. Finally, a positive exponent implies that the orbits are on a chaotic attractor, so the presence of a positive Lyapunov exponent indicates chaos. The Lyapunov exponents are defined as follows:

Consider two nearest neighboring points in phase space at time 0 and *t*, with distances of the points in the i^{th} direction $\|\delta x_i(0)\|$ and $\|\delta x_i(t)\|$, respectively. The Lyapunov exponent is then defined by the average growth rate λ_i of the initial distance,

$$\lambda_i=\lim_{t\to\infty}\frac{1}{t}\ln\frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|} \tag{5}$$

The existence of a positive Lyapunov exponent is the indicator of chaos showing neighboring points with infinitesimal differences at the initial state abruptly separate from each other in the i^{th} direction.[32] Using the algorithm introduced by Wolf et al., the Lyapunov exponent is calculated versus a given control parameter.[33] Then, the value of the control parameter increases a little and the Lyapunov exponent is calculated for the new control parameter. By continuing this procedure the Lyapunov exponent spectrum of the system is plotted versus the control parameter.

Bifurcation Diagrams

Bifurcation means a qualitative change in the dynamical behavior of a system when a parameter of the system is varied. A bifurcation diagram provides a useful insight into the transition between different types of motion that can occur as one parameter of the system alters. It enables one to study the behavior of the system on a wide range of an interested control parameter. In this paper the dynamical behavior of the system is studied through plotting the bifurcation diagrams of the normalized radius of the bubble versus different control parameters. The analysis of the bifurcation was carried out in the Poincare section *(P)*. To choose the Poincare section, we use the general method of setting one of the phase space coordinates to zero. In our analysis the condition:

$$P \equiv max_R\{(R, \dot{R}) : \dot{R} = 0\}$$

was used, which gives the maximal radii from each acoustic period. This condition was also used to draw the bifurcation diagram of a cavitation bubble in.[34] In order to obtain the bifurcation points, the equation of the bubble motion was solved numerically for 800 acoustic cycles of the lower frequency and a Poincare section was constructed. Considering only the last 200 cycles to make sure that the initial transient behavior is eliminated; for a given control parameter the points satisfying the above condition were plotted as $\frac{R}{R_0}$ in the bifurcation diagram. Then the control parameter was slightly increased and the new points were plotted versus the new control parameter. This procedure continued until the whole range of the interested control parameter was covered. For a full discussion about the Lyapunov exponent spectrum and bifurcation diagram and their utilization in order to study the bubble dynamics, one can refer to.[7-9, 25, 35] Lyapunov exponent and bifurcation diagrams have also been applied to study the dynamics of chemical systems. As an example, they have been applied as analysis tools to study the dynamics of the well-known Belousov-Zhabotinsky reaction.[36-40]

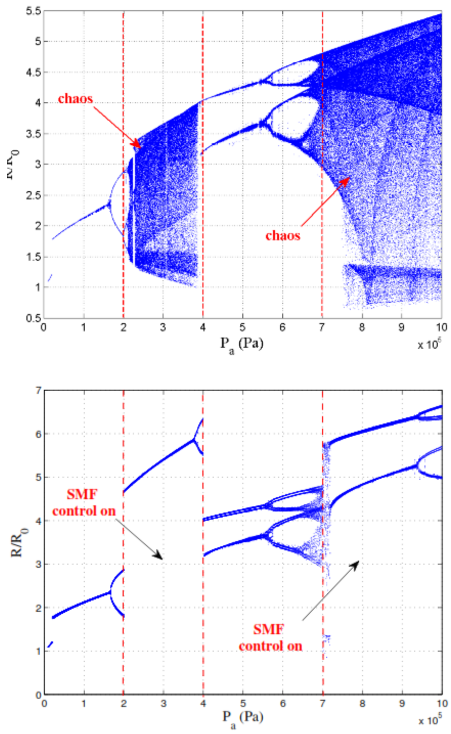

Fig. 1: Bifurcation diagrams of the normalized bubble radius driven by 300 kHz of frequency with the initial radius of 10 µm versus pressure: upper panel (a) chaotic behavior before applying the proposed technique, lower panel (b) after applying the Slave-Master Feedback control method.

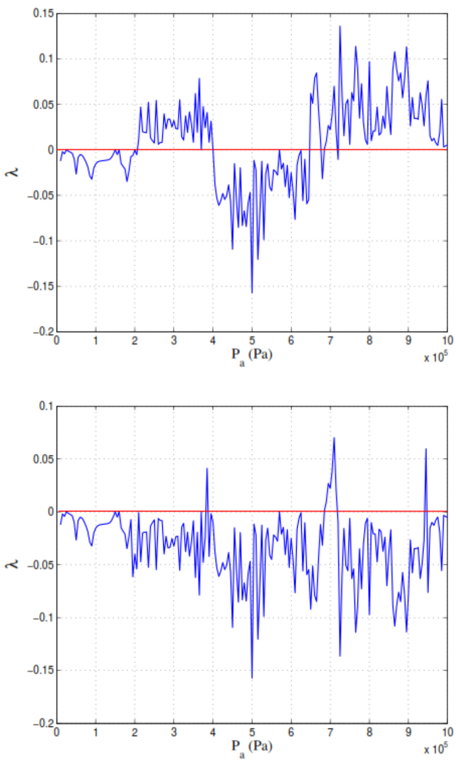

Fig. 2: Lyapunov spectra before and after applying the Slave-Master Feedback control method. Upper panel (a) Represents the case before applying the method while lower panel (b) represents the system after control.

RESULTS AND DISCUSSION

In order to streamline the manifestation of the method efficiency in suppressing chaos, some chaotic zones have been chosen as samples to be subjected to the Slave-Master Feedback control method. For the associated zones the dynamical behavior of the bubble was analyzed before and after control. This is done through computing its bifurcation diagram and the corresponding Lyapunov spectrum. Our goal is to seek maximum Lyapunov exponents and bifurcation analysis that can help us understand the dynamical behavior of the bubble with respect to the wide range of control parameters. Also time series of the normalized radius of the bubble are presented in order to reveal the stabilizing effect on the oscillations of the bubble in a given single value of the control parameter. The results are depicted in Figs. 1-8.

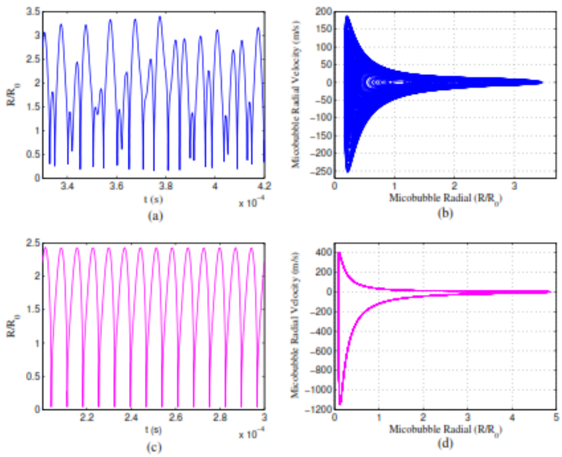

Fig. 3: Time series and trajectory in state space projection of normalized bubble radius driven by 10 µm initial radius and 250 kPa of pressure: (a) chaotic oscillations (Without applying the proposed technique), (b) regular oscillations (after applying the proposed technique).

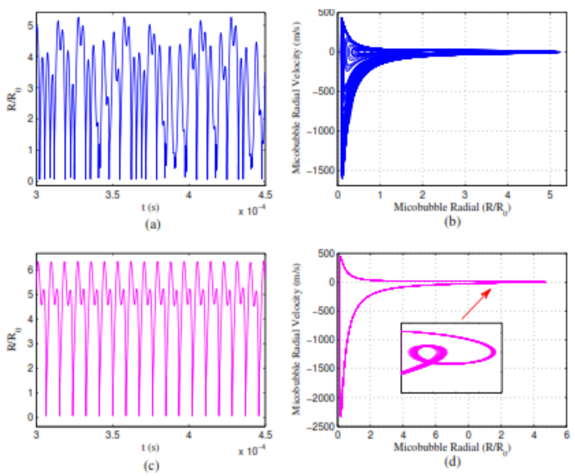

Fig. 4: Time series and trajectory in state space projection of normalized bubble radius driven by 10 µm initial radius and 900 kPa of pressure: (a) chaotic oscillations (Without applying the proposed technique), (b) regular oscillations (after applying the proposed technique).

The first sample (pressure-bifurcation diagram of bubble) is presented in Fig. 1(a). It belongs to a bubble with initial radius of 10 µm exposed to a single frequency force of 300 kHz when the control parameter is pressure in the range of 10 kPa - 1 MPa. In order to study the possibility of reducing chaos, a dynamical control method is applied. Fig. 1(b) presents the controlled dynamics after applying the dynamical control method. It is considerable that after applying the method, chaotic zone is reduced (see Fig. 1(b)). The maximum Lyapunov exponents is also an important indicator for a dynamical system to have a potentially chaotic behavior. Accordingly, the Lyapunov spectra is outlined in Fig. 2. Fig. 2(a) represents the original system, while Fig. 2(b) represents the controlled system, where Lyapunov exponent is mostly positive indicating a chaotic behavior. The existence of the negative Lyapunov exponent indicates the stable behavior. The controlling phenomenon has also been granted by plotting the normalized bubble oscillations versus time in a certain value of the pressure before and after control in Fig. 3 and 4.

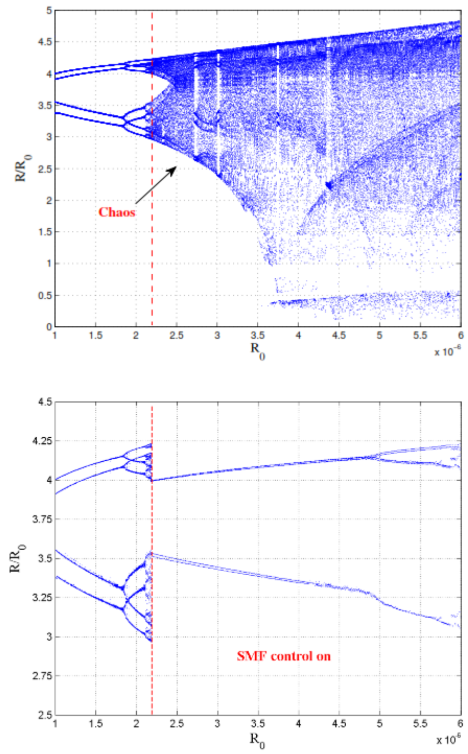

Fig. 5: Bifurcation diagrams of the normalized bubble radius driven by 300 kHz of frequency with the initial radius of 10 µm versus pressure: upper panel (a) chaotic behavior before applying the proposed technique, lower panel (b) after applying the Slave-Master Feedback control method.

Fig. 5(a) represents the second chaotic sample zone (radius-bifurcation diagram of bubble) before applying the dynamical control method. It belongs to a bubble subjected to a single frequency source of 300 kHz and amplitude 1 MPa, versus its initial radius as the control parameter. Fig. 5(b) shows the controlled dynamics. Also the control method is tested through the Lyapunov exponent diagrams (see Fig. 6). This figure indicates a significant abatement of the Lyapunov exponent from positive values to negative ones indicating that stable dynamics was achieved after the proposed technique was engaged. Fig. 6(a) corresponds the original system, and Fig. 6(b) corresponds the controlled system. Also the normalized oscillations of a bubble with 3 and 5 µm initial radius driven with 300 kHz of frequency and amplitude 1 MPa, before and after applying the Slave-Master Feedback control method, are shown in Fig. 7 and 8. The obtained results indicate that stable dynamics can be achieved after the proposed technique.

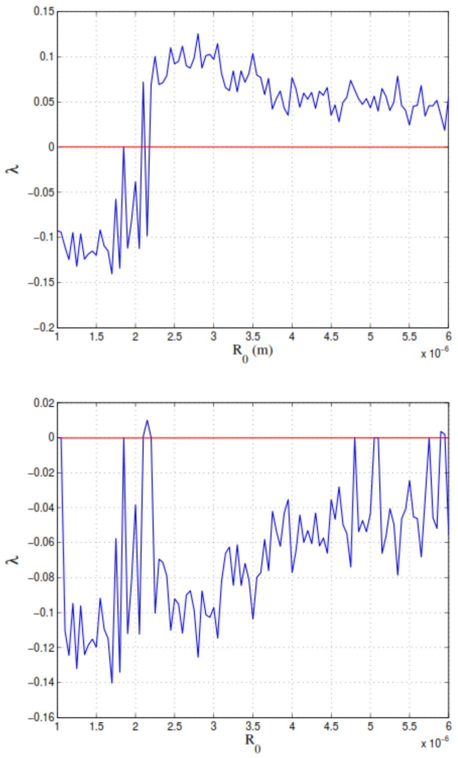

Fig. 6: Lyapunov spectra before and after applying the Slave-Master Feedback control method(initial radius of 10 µm). Upper panel (a) Represents the case before applying the method while lower panel (b) represents the system.

However, the possible role of Slave-Master Feedback in controlling the size of the bubble and hence deciding the optimum amplitude so as to minimize the energy expenditure was not addressed in previous studies. Therefore, it is necessary to have good understanding of the bubble dynamics, to provide reliable control mechanisms for the wide range applications in industry. A better understanding of cavitation bubbles' behavior is the first step toward controlling chaotic behavior of the bubble and using cavitation. Reducing chaos using Slave-Master Feedback control can be practically advantageous, in particular, in applications involving cavitation bubbles for medical purposes. However, chaotic oscillations of the bubbles decrease the treatment efficacy and makes it hard to control. Reducing chaotic dynamics can be a first step in increasing the predictability and safety of the treatment.

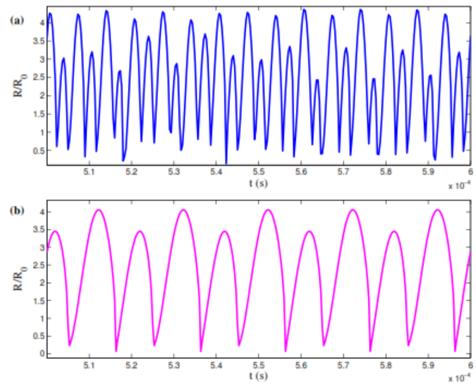

Fig. 7: Time series of normalized bubble radius driven by 3 µm initial radius, 300 kHz frequency and 1 MPa of pressure: (a) chaotic oscillations (Without applying the proposed technique), (b) regular oscillations (after applying the proposed technique).

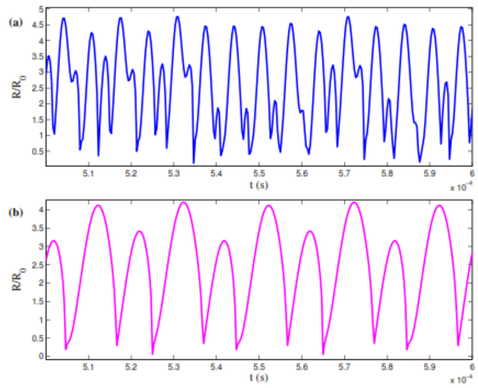

Fig. 8: Time series of normalized bubble radius driven by 5 µm initial radius, 300 kHz frequency and 1 kPa of pressure: (a) chaotic oscillations (Without applying the proposed technique), (b) regular oscillations (after applying the proposed technique).

CONCLUSION

The dynamics of acoustically driven gas bubbles has been studied applying the chaos physics methods.[7-9, 22] Results indicate its rich nonlinear and chaotic dynamics with respect to variations in the control parameters of the system. Hence, in the light of the above discussion, it can be stated that acoustic pressure shows its influence on the bubble dynamics.[7-9] Therefore, pressure can be regarded as the most important factor in stability of the bubble. From the practical point of view, frequency is the other important factor in the bubble dynamics. It is an essential factor, which can influence a spherical cavitation bubble dynamics. To reduce chaotic oscillations of the bubble, by considering frequency as a practical control tool, the new practical control method based on the frequency circuit is introduced. The bifurcation curves and Lyapunov exponent spectrum of the bubble output have shown that the chaotic dynamical behavior of the bubble can be totally converted to a stable state using the control method. This specific control scheme is of great importance since it is adaptable to applications involving acoustic cavitation phenomena. In fact, the physical application of the SMF method are in progress and the result will be reported elsewhere.[41] In order to accurately determine the control parameter values, one of the most important factors that should be taken in to account is the influence of the bubble-bubble interaction. Details of such clustering systems are very much important. This is because the bubble pulsation is strongly influenced by the interacting surrounding bubbles.[42] Therefor, due to the importance of topic in various aspects, investigation of the efficacy of the slave-master feedback control method in a system of interacting bubbles can be one of the subjects for future studies.

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